University of Puerto Rico
Río Piedras Campus
College of Natural Sciences
Department of Mathematics

Course Syllabus
First semester, academic year 2021-22

Course information

Course title: Computational Analysis I
Course code: MATE 6680
Credits/hours: 3 credits/45 hours
Course prerequisites: MATE 5201 (Advanced Calculus) and knowledge of a high level programming language.

Online due to the CoViD-19 emergency
Room: Virtual classroom via Google Meet, code: ofs-aguc-yvj
Time: Monday and Wednesday 4:00-5:20 pm
Course webpage: Moodle site

Instructor: Mariano Marcano
Office: Virtual via Google Meet, code: rgj-ckqr-qtw
Email address: mariano.marcano@upr.edu
Office hours: Monday 11:00 am-1:00 pm and Wednesday 1:00-2:00 pm or by appointment.
Professor's webpage: epsilon.uprrp.edu/mmarcano

Course description

Numerical analysis aims to provide computational methods to study and solve mathematical problems involving real variables. Because the methods provide approximations to the true solution of the problem, the study of errors is very important to numerical analysis. In this course we will: provide the mathematical foundations of numerical methods; analyze the method basic theoretical properties–stability, accuracy, and computational complexity; and illustrate the method performance by means of computational examples and counterexamples by using a high level programming language, e.g., MATLAB®/Octave.

Course objectives

After completing the course students will know how to:

1. analyze equation conditioning, stability, and accuracy;
2. solve large systems of linear equations by using the computer;

3. use methods of factorization, orthogonalization, and triangularization of matrices and apply them to solve linear systems, including least-squares problems;
4. solve eigenvalue problems numerically;
5. use numerical methods to approximate the solutions of nonlinear systems of equations and understand the conditions that a system must meet to ensure the convergence of the method;
6. use polynomial interpolation to approximate the values of a function and analyze the error of such methods;
7. integrate numerically a function.

Content outline and schedule

	Topic	Time
1.	Review of linear algebra.	3 hours
2.	Basic concepts of numerical analysis and the floating point representation of real numbers. (Asynchronous)	3 hours
3.	The direct solution of linear systems: Gaussian elimination and matrix decomposition: (a) LU decomposition (1.5 hour); (b) Gaussian elimination with pivoting (1.5 hour); (c) The Cholesky and tridiagonal systems methods (3 hours).	6 hours
4.	Stability and error analysis of linear systems	3 hours
5.	Iterative methods to approximate the solution of linear systems: (a) Method derivation (1.5 hour) (b) Method convergence analysis (1.5 hour).	3 hours
6.	Geometrical location and numerical approximations of eigenvalues: (a) Gerschgorin Theorem (0.5 hour); (b) The power method (1 hour); (c) The QR decomposition (3 hours).	4.5 hours
7.	Least-squares problems	1.5 hour
8.	Exam 1 (Wednesday, October 20, in person)	1.5 hour
9.	Numerical solution of nonlinear equations (a) Root finding of nonlinear equations (1.5 hour) (b) Solution of systems of nonlinear equations by iterative methods: numerical methods and their convergence analysis (4.5 hours).	6 hours

10. Polynomial interpolation: 6 hours

    (a) Lagrange interpolation (1.5 hour);
    (b) Newton form of the interpolating polynomial (1.5 hour);
    (c) Piecewise interpolation by using splines: Formula derivation, examples, and error bounds (3 hours).

11. Numerical integration: 6 hours

    (a) Quadrature formula (1.5 hour);
    (b) Newton-Cotes formulas (1.5 hour);
    (c) Composite Newton-Cotes formula (1.5 hour);
    (d) Automatic integration (1.5 hour).

12. Exam 2 (Monday, December 6, in person) 1.5 hour

Total: 45 hours

Instructional strategies

- The course will be taught online. The content of the course and supporting materials will be available to the student through the Moodle platform. The lecture notes will be in slide presentations. Video conferences will be conducted by Google Meet at the time of the course to discuss the class topics, use code: ofs-aguc-yvj
- Assignments will be uploaded to Moodle in pdf format, the students will download the files and, before the deadline, the students must upload the answers to Moodle in a single pdf file. Grading and feedback will be available in Moodle to the student.
- A frequent asked question FAQ Moodle forum will be available to post any question the student may have and answer questions posted by classmates. One point bonus (maximum two points) will be added to the corresponding homework of any student that answers completely and correctly a question of the forum.

Available and required learning resources

To take advantage of the course material and other resources in Moodle the student needs a personal computer with fast internet access, a PDF viewer, an internet browser, a word processor (LaTeX is free and suitable for writing mathematical expressions), and the high level programming platform MATLAB®/Octave. Octave is free and can be downloaded from here.

Course evaluation

Evaluation of student understanding of the class material will be made by means of homework assignments and in-class exams. The grade will be computed as follows:

Homeworks	40%
Exam 1	30%
Exam 2	30%
	100%

Assignments will be posted in Moodle at least one week before the deadline.

Grading system

Letter system (A, B, C, D or F).

Law 51: Rights of Students with Disabilities

Students with access to Vocational Rehabilitation Services should contact the professor at the beginning of the semester in order to plan any special arrangements and equipment necessary in accordance with the recommendations of the Office of Challenged Students' Affairs (OAPI) in the Office of the Dean of Students. In addition, any students with special needs or who require any type of assistance or special arrangements should contact the professor.

Academic Integrity

The University of Puerto Rico promotes the highest standards of academic and scientific integrity. Article 6.2 of the UPR Students General Bylaws (Board of Trustees Certification 13, 2009- 2010) states that academic dishonesty includes, but is not limited to: fraudulent actions; obtaining grades or academic degrees by false or fraudulent simulations; copying the whole or part of the academic work of another person; plagiarizing totally or partially the work of another person; copying all or part of another person answers to the questions of an oral or written exam by taking or getting someone else to take the exam on his/her behalf; as well as enabling and facilitating another person to perform the aforementioned behavior. Any of these behaviors will be subject to disciplinary action in accordance with the disciplinary procedure laid down in the UPR Students General Bylaws.

Normativeness on discrimination by sex and gender in sexual violence form

The University of Puerto Rico prohibits discrimination based on sex, sexual orientation, and gender identity in any of its forms, including that of sexual harassment. According to the Institutional Policy Against Sexual Harassment at the University of Puerto Rico, Certification Num. 130, 2014-2015 from the Board of Governors, any student subjected to acts constituting sexual harassment, must come to the Office of the Student Ombudsperson, the Office of the Dean of Students, and/or the Coordinator of the Office of Compliance with Title IX for an orientation and/or a formal complaint.

Bibliography

Textbook: Alfio Quarteroni, Riccardo Sacco, and Fausto Saleri. *Numerical Mathematics (Texts in Applied Mathematics , Vol. 37)* Second Edition, 2007. ISBN 978-3-540-49809-4. Available online from the UPR library web page.

References:

1. Kendall E. Atkinson, *An Introduction to Numerical Analysis*, Second Edition, John Wiley & Sons, 1989.

2. Eugene Isaacson and Herbert B. Keller, *Analysis of Numerical Methods*, Dover Publications, Inc., New York, 1994.

3. James M. Ortega, *Numerical Analysis: A Second Course*, SIAM. Philadelphia, 1990.

4. James M. Ortega and Werner C. Rheinboldt, *Iterative Solutions of Nonlinear Equations in Several Variables*, Academic Press, Inc., San Diego, CA, 1970.

5. G. W. Stewart, *Introduction to Matrix Computations*, Academic Press, Inc., San Diego, CA, 1973.

6. J. Stoer and R. Bulirsch, *Introduction to Numerical Analysis (Texts in Applied Mathematics, No 12)*, Second edition, Springer, NY, 1997.

7. Lloyd N. Trefethen and David Bau III, *Numerical Linear Algebra*, SIAM, Philadelphia, 1997.

Electronic references

1. M. Marcano, *Introduction to* MATLAB, 2007, preprint.

2. Steven Johnson. 18.335J Introduction to Numerical Methods . Spring 2019. Massachusetts Institute of Technology: MIT OpenCourseWare, License: Creative Commons BY-NC-SA.

3. I. Rubio, *CCOM introduction to LATEX*, 2019, preprint.

4. LN Trefethen, *The definition of numerical analysis*, 1992, Essay.